CCGPS Frameworks Student Edition

Mathematics

CCGPS Coordinate Algebra

Unit 3: Linear and Exponential Functions

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"Making Education Work for All Georgians"

Unit 3
Linear and Exponential Functions

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OVERVIEW

In this unit student will:

- Represent and solve linear equations and inequalities graphically using real-world contexts.
- Use function notation.
- Interpret linear and exponential functions that arise in applications in terms of the context.
- Analyze linear and exponential functions and model how different representations may be used based on the situation presented.
- Build a function to model a relationship between two quantities.
- Create new functions from existing functions.
- Construct and compare linear and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Represent and solve equations and inequalities graphically
MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on*

linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)

MCC9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.★

Understand the concept of a function and use function notation

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of *f* is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

Interpret functions that arise in applications in terms of the context

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.★ *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★ *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

Analyze functions using different representations

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima. ★

MCC9-12.F.IF.7e Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~ ★

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. ★ *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

Construct and compare linear, ~~quadratic,~~ and exponential models and solve problems

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. ★

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. ★

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~ ★

Interpret expressions for functions in terms of the situation they model

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. ★ *(Limit exponential functions to those of the form f(x) = bx + k.)*

RELATED STANDARDS

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity

of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear and exponential functions that arise in applications in terms of the context.

- When analyzing linear and exponential functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- New functions can be created from existing functions.
- Understand how to construct and compare linear and exponential models and solve problems.
- Understand how to interpret expressions for functions in terms of the situation they model.

CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained:

- Know how to solve equations, using the distributive property, combining like terms and equations with variables on both sides.
- Know how to solve systems of linear equations.
- Understand and be able to explain what a function is.
- Determine if a table, graph or set of ordered pairs is a function.
- Distinguish between linear and non-linear functions.
- Write linear equations and use them to model real-world situations.

SELECT TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children.
Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Arithmetic Sequence.** A sequence of numbers in which the difference between any two consecutive terms is the same.

- **Average Rate of Change.** The change in the value of a quantity by the elapsed time. For a function, this is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph.

- **Coefficient.** A number multiplied by a variable in an algebraic expression.

- **Constant Rate of Change.** With respect to the variable x of a linear function $y = f(x)$, the constant rate of change is the slope of its graph.

- **Continuous.** Describes a connected set of numbers, such as an interval.

- **Discrete.** A set with elements that are disconnected.

- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.

- **Explicit Expression.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.

- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in $y = ab^x$.

- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.

- **Even Function.** A function with a graph that is symmetric with respect to the *y*-axis. A function is only even if and only if $f(–x) = f(x)$.

- **Factor.** For any number x, the numbers that can be evenly divided into x are called factors of x. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.

- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.

- **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

- **Linear Function.** A function with a constant rate of change and a straight line graph.

- **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Odd Function.** A function with a graph that is symmetric with respect to the origin. A function is odd if and only if $f(-x) = -f(x)$.

- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

- **Range.** The set of all possible outputs of a function.

- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of a_n.

- **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

- **Term.** A value in a sequence--the first value in a sequence is the 1^{st} term, the second value is the 2^{nd} term, and so on; a term is also any of the monomials that make up a polynomial.

- **Vertical Translation.** A shift in which a plane figure moves vertically.

- ***X*-intercept.** The point where a line meets or crosses the *x*-axis

- ***Y*-intercept.** The point where a line meets or crosses the *y*-axis

Talk is Cheap!

Mathematical Goals

- Graph linear functions
- Use the graphing calculator to find the intersection of two linear functions
- Interpret the intersection in terms of the problem situation
- Compare functions represented algebraically, graphically, and in tables

Common Core State Standards

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.★ *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Model with mathematics.
3. Use appropriate tools strategically.

Introduction

This task can be used to introduce students to functions in a realistic setting—choosing a cell phone plan given certain conditions. Students gain experience working with decimals and translating among different representations of linear functions. They use the graphing calculator to find the intersection of two linear functions graphically and interpret the intersection in terms of the problem situation.

Materials

- Pencil
- Handout
- Graphing calculator

To encourage communication between parents and their children and to prevent children from having extremely large monthly bills due to additional minute charges, two cell phone companies are offering special service plans for students.

Talk Fast cellular phone service charges $0.10 for each minute the phone is used.
Talk Easy cellular phone service charges a basic monthly fee of $18 plus $0.04 for each minute the phone is used.

Your parents are willing to purchase for you one of the cellular phone service plans listed above. However, to help you become fiscally responsible they ask you to use the following questions to analyze the plans before choosing one.

1. How much would each company charge per month if you talked on the phone for 100 minutes in a month? How much if you talked for 200 minutes in a month?

2. Build a table, make a graph, and write a function rule, $f(x)$ or $g(x)$, to represent the cost of each cellular service in terms of the number of minutes, x.

<u>Table:</u>

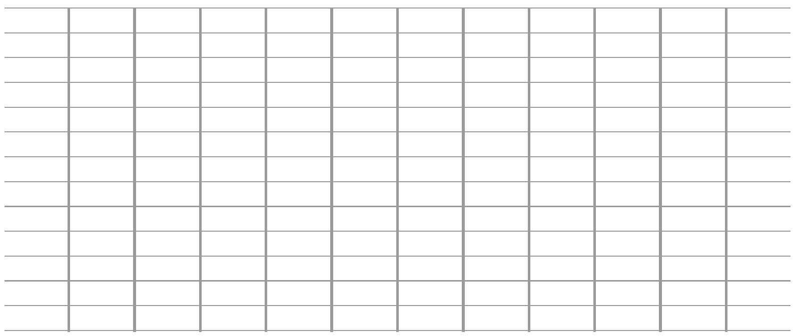
<u>Talk Fast:</u>

(numbers of minutes) x								
(cost in dollars) $f(x)$								

Talk Easy:

(numbers of minutes) ***x***								
(cost in dollars) ***g(x)***								

Graph:



Rule:

Use the table, graph, and/or rule to help answer the following questions:

3. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.

4. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.

5. Depending on the number of minutes you talk on the phone each month, explain to your parents which cellular phone plan is more economical. Include in your explanation the point at which both cellular phone plans cost the same amount of money.

6. If you know the cost of each plan for 300 minutes, can you double this cost to find the cost for 600 minutes? Explain your answer.

Functioning Well

Mathematical Goals

- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Common Core State Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★ *(Focus on linear and exponential functions.)*

Standards for Mathematical Practice

1. **Look for and make use of structure.**
2. **Look for and express regularity in repeated reasoning.**

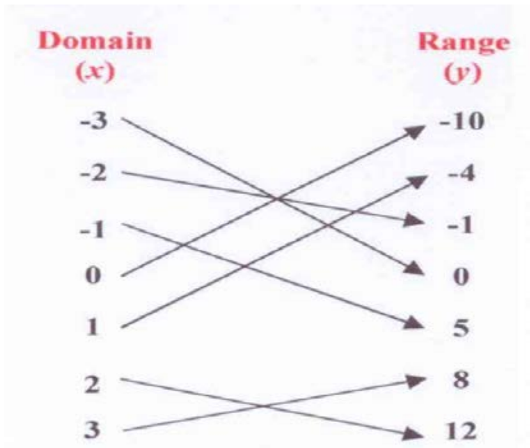
Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Function or Not

Determine whether or not each of the following is a function or not. Write function or not a function and explain why or why not.

Relation Answer and Explanation

1.



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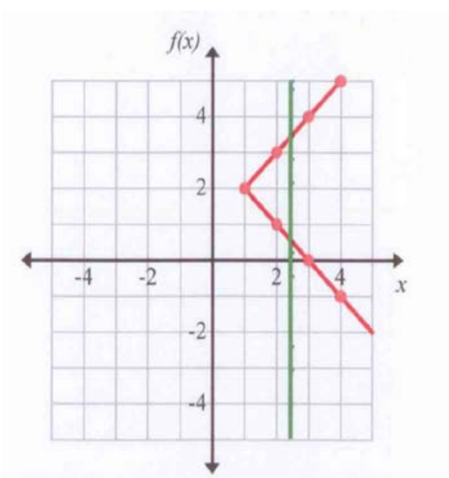
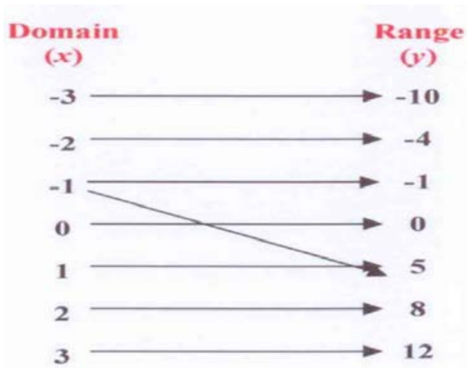
2.



Relation   Answer and Explanation

3.



4. *(x*, *y*) = (*student's name, shirt color*)

Function Notation

Use the scenario to answer the questions below.

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E + 15)$ and $f(E) + 15$ mean?

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

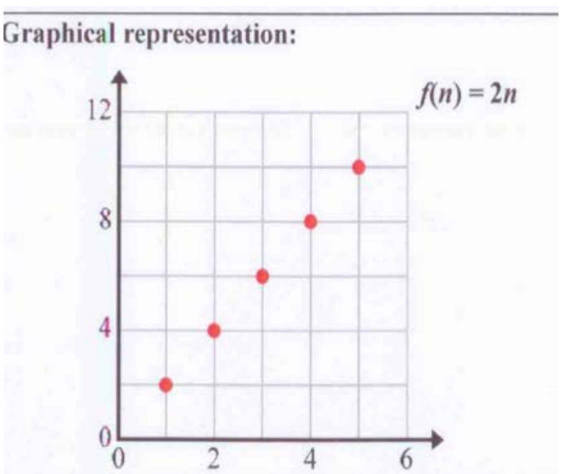
7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for 2 nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents for dinner at the restaurant. Illustrate using function notation

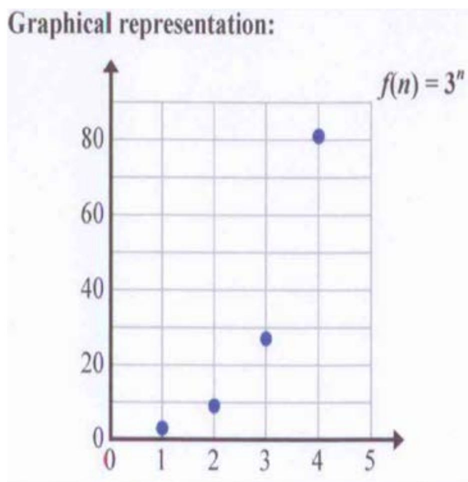
Sequences are Functions

Write an explicit and a recursive formula to represent the graphs below.

9.



10.



<u>You're Toast, Dude!</u>

Mathematical Goals

- Use function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities

Common Core State Standards

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.★

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.★ *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.★

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.★ *(Limit to linear and exponential functions.)*

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★ *(Limit exponential functions to those of the form $f(x) = bx + k$.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Look for and make use of structure.
4. Model with mathematics.
5. Use appropriate tools strategically.

Introduction

Students extend their understanding of functions. Students will gain experience in moving between a problem context and its mathematical model in order to solve problems and make decisions.

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400 and the cost of producing each toaster is an additional $4 per toaster.

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

2. What is the total cost of producing 100 toasters in one week?

3. If you produce 100 toasters in one week, what is the total production cost per toaster?

4. Will the total production cost per toaster always be the same? Justify your answer.

5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:

a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

c. How many toasters must be produced to have a total production cost per toaster of $8?

<u>Community Service, Sequences, and Functions</u>

Mathematical Goals

- **Recognize that sequences are functions sometimes defined recursively**
- **Use technology to graph and analyze functions**
- **Convert a recursive relationship into an explicit function**
- **Construct linear and exponential function (including reading these from a table)**
- **Observe the difference between linear and exponential functions**

Common Core State Standards

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. ★ *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★ *(Focus on linear and exponential functions.)*

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. ★

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. ★

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~ ★
MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

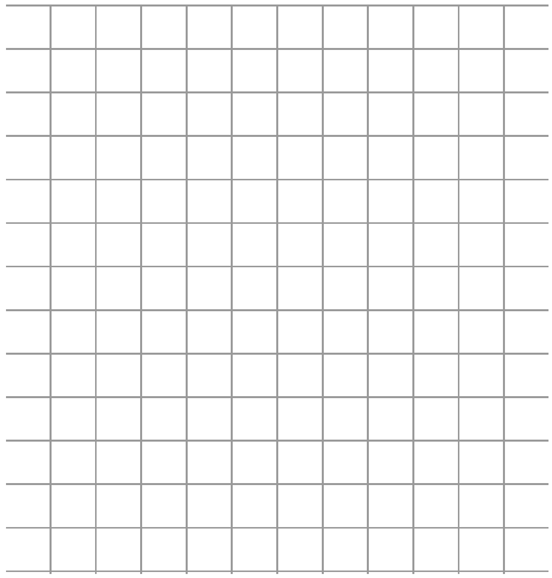
Standards for Mathematical Practice
1. Reason abstractly and quantitatively.
2. Model with mathematics.
3. Use appropriate tools strategically.
4. Look for and make use of structure.
5. Look for and express regularity in repeated reasoning.

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the table below for the Great Seven Day Cleanup.

X	**Y**
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

2. Sketch the data on the grid below and use the graphing calculator precision.



3. Determine type of function modeled in the graph above and describe key features of the graph.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

5. Using the same data gathered during the Great Four Day Cleanup, write an explicit formula using function notation.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days and was renamed the Great Seven Day Cleanup?

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decided to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decided to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects for their next community service project. Assuming there was a total of 10 customers on the first day; you will also need to determine a recursive formula and an explicit formula for the number of pies sold and the price of the pies for the n^{th} customer.

a. On the first day of selling pies, each customer buys the same number of pies as his customer number. Complete the table.

Customer Number	Number of Pies Sold	Cost of Pie(s)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

b. Write a recursive and explicit formula for the pies sold on day one. Explain your thinking.

c. On the second day of selling pies the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established.

Customer Number	Number of Pies Sold	Cost of Pie(s)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
total		

d. Write a recursive and explicit formula for the pies sold on day two. Explain your thinking.

8. Compare the pies sold and the amount earned from the pies on day one to that of day two (compare the situations modeled and use key features of the functions modeled to make your comparison).

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.

Building and Combining Functions

Mathematical Goals

- Calculate and interpret rate of change
- Combine functions
- Write explicit function rules

Common Core State Standards

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.★

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.★ *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice
1. Reason abstractly and quantitatively.
2. Look for and make use of structure.
3. Model with mathematics.
4. Use appropriate tools strategically.

Combining Functions

Given the functions $f(x) = 4x - 5$ and $g(x) = 3^x$

1. Find $f(x) + g(x)$

2. Find $\dfrac{f(x)}{g(x)}$

The number of violent crimes committed in major cities is one statistics that is used to determine the safety rating of that city. In this task, we will examine data from two cities to not only make conclusions about those cities, but to examine the relationships of the crime rates to other factors relative to each city. In table 1, the number of violent crimes committed in each city is given by year. In table 2, the population of each city is given by year.

TABLE 1:

Year	2000	2001	2002	2003	2004	2005
City A	793	795	807	818	825	831
City B	448	500	525	566	593	652

3. By just looking at the raw data for the number of crimes, which city would you predict is safer? Why?.

Year	2000	2001	2002	2003	2004	2005
City A	61,000	62,100	63,220	64,350	65,510	66,690
City B	28,000	28,588	29,188	29,801	30,427	31,066

4. By just looking at the raw data for the population, which city would you predict is safer? Why?

5. Do you think that these two data sets could be related? How? Why?

Let's define functions to represent the data as we have it. Let $C(t)$ be the function that represents the number of crimes in t year, where t is measured in number of years since 2000. That means that for city A, $C(0) = ?$, $C(1) = ?$ $C(4) = ?$

Let $P(t)$ be the function that represents the population in t year, where t is measured in number of years since 2000. That means that for city A, $P(0) = ?$, $P(1) = ?$, $P(4) = ?$

6. We have just identified another notational issue. How will we know to which city we are referring?

7. Since the independent variable in our data is time, notice that each function written is dependent upon time. That means for us to find the per capita crime rate for each city, that is to compare the number of crimes to the number of people we need the ratio of these two functions. Let $R_A(t)$ be the per capita crime rate in city A and $R_B(t)$ be the per capita crime rate in city B. Using $C(t)$ and $P(t)$ for the appropriate cities, write the functional rule for $R(t)$.

8. Now that you have the two functions defined, complete the table below showing the per capita violent crime rate in both cities by year using the data from Table 1 and 2. Write each of the function values as percents.

***t* (years)**	2000	2001	2002	2003	2004	2005
$R_A(t)$						
$R_B(t)$						

9. Now, using this data, which city is safer? Why?

10. Make any conclusions about the trends you see in the data. What did you base your conclusion on?

11. Write a function rule for $C_A(t)$, $C_B(t)$, $P_A(t)$, $P_B(t)$, and then using these function rules, write an explicit function rule for $R_A(t)$ and $R_B(t)$. Verify that each function gives the correct value that you calculated from the data in the table above.

12. Using the functions, can you make predictions about crime rates in the future if the trends in the given data continue?

Given the functions *f* and *g* as defined in the table below

X	f(x)	g(x)
1	3	2
2	4	1
3	1	4
4	2	3

Complete the tables for the following functions:

13. $n(x) = f(x) + g(x)$ What kind of function is n(x)? Why?

X	f(x)	g(x)	n(x)

14. $p(x) = 2f(x)g(x) - f(x)$

X	**f(x)**	**g(x)**	**p(x)**

15. q(x) = g(x)/f(x)

X	**f(x)**	**g(x)**	**q(x)**

adapted from Functions Modeling Change, A Preparation for Calculus, Connally, Hughes-Hallett, Gleason, et al. John Wiley & Sons, 1998.

<u>High Functioning!</u>

Mathematical Goals

- Find the value of k given the graphs
- Identify even and odd function
- Relate vertical translations of a linear function to its *y*-intercept

Common Core State Standards

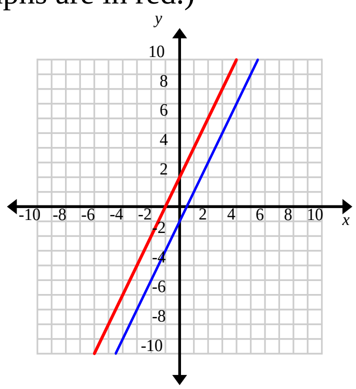
MCC9-12.F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

Standards for Mathematical Practice

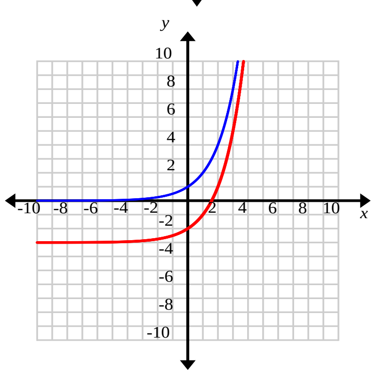
1. Reason abstractly and quantitatively.
2. Look for and make use of structure.
3. Model with mathematics.
4. Attend to precision.

Given the graph and the original function, find k and the new function. Describe the translation. (The original graphs are in blue and the transformed graphs are in red.)

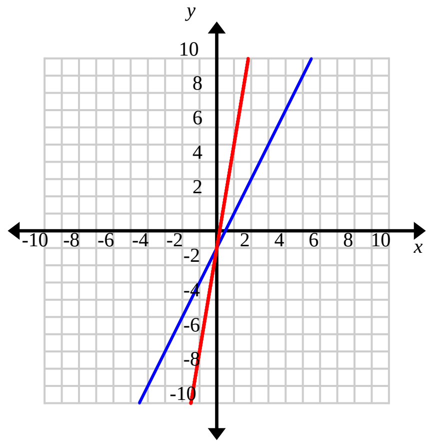
1. $f(x) = 2x - 1$



2. $g(x) = 2^x$



3. $f(x) = 2x - 1$



Consider the relationship between Fahrenheit and Celsius temperatures. Using your graphing calculator, graph these two functions on the same set of axes:

$$y = x \text{ and } y = \frac{5}{9}(x - 32)$$

4. Describe in transformational terms, how the first graph becomes the second graph.

5. At what temperature are the Fahrenheit and Celsius readings the same?

6. Using the definitions of odd and even functions, determine if the graphs and algebraic expressions are odd, even, or neither. Justify your answer.

a.

$$h(x) = x^3$$
$$-h(x) = -x^3$$
$$h(-x) = (-x)^3 = -x^3$$

b.

$$g(x) = 4^x$$

$$g(-x) = 4^{-x} = \frac{1}{4^x}$$

c.